

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2022

17/18UMT1MC01 – ALGEBRA AND CALCULUS - I

Date: 24-11-2022

Dept. No.

Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

PART – A

Answer ALL the Questions:

(10 × 2 = 20)

1. Find the n^{th} derivative of $\cos(ax + b)$.
2. Give the formulae for subtangent and subnormal in cartesian form.
3. Define saddle point.
4. List out the conditions for a function $u = f(x, y)$ to have maxima.
5. What is the formula to find the radius of curvature in cartesian coordinates?
6. Write the coordinates of centre of curvature.
7. Frame the quadratic equation, one of whose roots is $1 + i$.
8. Determine the product of the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$.
9. How to eliminate the coefficient of x^2 from a cubic equation?
10. Find the number of positive roots of the equation $x^3 - 6x^2 + 13x - 10 = 0$.

PART – B

Answer any FIVE of the following:

(5 × 8 = 40)

11. Find the angle of intersection of the cardioids $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$.
12. Find maxima and minima of the function $2(x^2 - y^2) - x^4 + y^4$.
13. What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1,1)?
14. Find the asymptotes of the cubic equation $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$.
15. Discuss the nature of the roots of the equation $3x^5 - 2x^3 - 4x + 2 = 0$.
16. Solve the equation $x^3 - 27x + 54 = 0$ using Cardon's method.
17. Calculate the sum of the cubes of the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$.
18. Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetical progression if $2p^3 - 9pq + 27r = 0$.

PART – C

Answer any TWO of the following:

(2 × 20 = 40)

19. a) If $y = a \cos(\log x) + b \sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(10 + 10)**

b) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

20. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(20)**

21. a) Solve the reciprocal equation $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$. **(10 + 10)**

b) Increase the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$ by 7.

22. Using Horner's method, find the real root of $x^3 - 4x + 2 = 0$ correct to 3 places of decimal. **(20)**

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